Correcting wide-band images for primary beam effects

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In aperture synthesis imaging, the technique of multi-frequency synthesis (MFS) (Conway 1990) is used to combine measurements over a wide bandwidth and to account for a frequency-dependent sky brightness. A common parameterisation of the sky brightness is a modified power law,

$$I(\nu) = I(\nu_0) \left(\frac{\nu}{\nu_0}\right)^{\alpha+\beta \log(\nu/\nu_0)} \tag{1}$$

where α represents the spectral index at the reference frequency, ν_0, and β represents the spectral curvature, *i.e.*. the change in spectral index per e-folding frequency.

In many implementations of MFS imaging, the sky brightness is linearised *via* a Taylor expansion of the form

$$I(\nu) = \sum_t I_t \left(\frac{\nu - \nu_0}{\nu_0}\right)^t \tag{2}$$

The first three terms of this series, written out explicitly below,

$$\mathrm{tt0} \equiv I_{t=0} = I(\nu_0) \tag{3a}$$

$$\mathrm{tt1} \equiv I_{t=1} = \alpha\, I(\nu_0) \tag{3b}$$

$$\mathrm{tt2} \equiv I_{t=2} = [\beta + \alpha(\alpha - 1)/2]\, I(\nu_0) \tag{3c}$$

are often adequate for typical bandwidth ratios.

However, wide-field images are also affected by the telescope's frequency-dependent primary beam, for which an image-plane correction is often required. A convenient way to handle these corrections is to parameterise the primary beam, $P(\nu)$, in the same manner used for the sky (*i.e.*, as in Equation 1),

$$I(\nu) = I_s(\nu)\, P(\nu) \tag{4a}$$

$$= I_s(\nu_0)\, P(\nu_0) \left(\frac{\nu}{\nu_0}\right)^{[\alpha_p+\alpha_s]+[\beta_p+\beta_s] \log(\nu/\nu_0)} \tag{4b}$$

which results in a simple relationship between the apparent properties $(I(\nu), \alpha, \beta)$, the true sky's properties $(I_s(\nu), \alpha_s, \beta_s)$ and the primary beam's response $(I_p(\nu), \alpha_p, \beta_p)$,

$$I(\nu_0) = I_s(\nu_0)\, P(\nu_0) \tag{5a}$$

$$\alpha = \alpha_s + \alpha_p \tag{5b}$$

$$\beta = \beta_s + \beta_p \tag{5c}$$

(See Rau (2011) for a more complete discussion).

If the primary beam is not accounted for during gridding then the Taylor term images will absorb both the sky and primary beam response, as can be seen by substituting Equations 5 into Equations 3,

$$\mathrm{tt0} = I_s(\nu_0)\, P(\nu_0) \tag{6a}$$

$$\mathrm{tt1} = (\alpha_s + \alpha_p)\, I_s(\nu_0)\, P(\nu_0) \tag{6b}$$

$$\mathrm{tt2} = [(\beta_s + \beta_G) + (\alpha_s + \alpha_p)(\alpha_s + \alpha_p - 1)/2]\ I_s(\nu_0)\, P(\nu_0) \tag{6c}$$

With an appropriately parameterised model of the primary beam, the effects of the primary beam can be removed to produce a set of 'primary beam corrected' Taylor term images (denoted below with a prime) that only encode information about the sky brightness.

$$\mathrm{tt0}' = I_s(\nu_0) = P^{-1}(\nu_0)\,\mathrm{tt0} \tag{7a}$$

$$\mathrm{tt1}' = \alpha_s\, I_s(\nu_0) = P^{-1}(\nu_0)\,(\mathrm{tt1} - \mathrm{tt0}\,\alpha_p) \tag{7b}$$

$$\mathrm{tt2}' = [\beta_s + \alpha_s(\alpha_s - 1)/2]\, I_s(\nu_0) = P^{-1}(\nu_0)\,(\mathrm{tt2} - \mathrm{tt1}\,\alpha_p - \mathrm{tt0}[\beta_p - \alpha_p(\alpha_p + 1)/2]) \tag{7c}$$

Note that the common leading factor of $1/P(\nu_0)$ means that these images can also be processed by a standard linear mosaic algorithm, although it remains to be shown whether this produces the optimum weighting for the $t > 0$ Taylor terms.

A second topic of interest is the parameterisation of the primary beam. A generic solution would be to obtain a spectral cube of the primary beam's power pattern and fit Equation 1 to each line of sight. However, it may be preferable to derive closed-form expressions from analytical beam models. Sault (1994) demonstrate that such a model has an effective beam spectral index of

$$\alpha_p \equiv \frac{\nu}{P}\frac{\partial P}{\partial \nu} \tag{8}$$

and this technique can be extended to calculate an effective beam curvature,

$$\beta_p \equiv \frac{\nu^2}{2P}\frac{\partial^2 P}{\partial \nu^2} - \frac{\alpha_p(\alpha_p - 1)}{2} \tag{9}$$

In the specific case of a Gaussian beam model of the form

$$P_G(\theta,\nu) = \exp\left(-4\log(2)\left(\frac{\theta}{\theta_0}\right)^2\left(\frac{\nu}{\nu_0}\right)^2\right) \tag{10}$$

where θ_0 is the FWHM at the reference frequency, Equation 8 yields

$$\alpha_{p_G} = -8\log(2)\left(\frac{\theta}{\theta_0}\right)^2\left(\frac{\nu}{\nu_0}\right)^2 \tag{11}$$

and Equation 9 evaluates to

$$\beta_{P_G} = \alpha_{p_G} = -8\log(2)\left(\frac{\theta}{\theta_0}\right)^2\left(\frac{\nu}{\nu_0}\right)^2 \tag{12}$$

Note that while these equations are generic enough to calculate the beam spectral index and curvature away from the reference frequency, they should be evaluated at the reference frequency when used together with Equation 7.

REFERENCES

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