Direction to magnetic source analysis of single string of down-hole magnetic tensor data

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SUMMARY

Eigen-analyses of the magnetic gradient tensor provide powerful tools for characterising and targeting magnetic bodies. A very useful invariant known as the scaled moment or normalised source strength peaks directly over a source body for a number of source geometries. Using this invariant, we investigate an angular minimisation method to triangulate the direction to source using a single string of data measured down-hole with no assumption about the source geometry or position. The results are excellent for constraining the centre of magnetisation for the ideal case of an isolated dipole. When applied to multiple sources or non-dipolar geometries the observation string should be post-analysed as a series of smaller observation strings. Then, the estimated directions to source pass through the source body or bodies. Gradient tensor analyses, such as those presented here, will continue to become more important as new gradiometer systems start to come online.

INTRODUCTION

In mineral exploration many magnetic targets are drilled using information obtained through the interpretation and inversion of total magnetic intensity (TMI) data at the surface. TMI data measured down-hole provides access for more proximal measurements and carries richer information, however, there are severe limitations to the interpretation of a single string of TMI data. Instead, measurements of the gradients of the magnetic field components provides five independent pieces of information at each observation point. This compensates for the limited spatial distribution of measurements and, together with the increased proximity to the source, yields much superior targeting capabilities.

In anticipation of the development of down-hole gravity gradiometers Rim and Li (2010) recently showed that down-hole tensor targeting methods are feasible, especially when applied to a single compact source. Clark (2012) applied eigen-analyses and deconvolution to magnetic line data at Mount Leyshon, in Queensland Australia, to obtain directions to equivalent sources of different geometries. The aim of the present paper is to extract directions to synthetic sources from the convergence of eigenvector solutions without any assumption of source geometry.

THEORY

The magnetic gradient tensor

In the absence of currents, the magnetic field B becomes irrotational i.e., \[ \nabla \times \mathbf{B} = 0, \] and then may be written in terms of a scalar potential \( V \)

\[ \mathbf{B} = -\nabla V. \] The directional derivatives of the three components of the magnetic field form the magnetic gradient tensor \( \Gamma \) which may be written as a 3 \( \times \) 3 matrix,

\[
\begin{pmatrix}
\frac{-\partial^2 V}{\partial x^2} & \frac{-\partial^2 V}{\partial x \partial y} & \frac{-\partial^2 V}{\partial x \partial z} \\
\frac{-\partial^2 V}{\partial y \partial x} & \frac{-\partial^2 V}{\partial y^2} & \frac{-\partial^2 V}{\partial y \partial z} \\
\frac{-\partial^2 V}{\partial z \partial x} & \frac{-\partial^2 V}{\partial z \partial y} & \frac{-\partial^2 V}{\partial z^2}
\end{pmatrix},
\]

\begin{align*}
\Gamma &= \mathbf{V} \mathbf{B}, \\
&= \begin{pmatrix}
B_{xx} & B_{xy} & B_{xz} \\
B_{yx} & B_{yy} & B_{yz} \\
B_{zx} & B_{zy} & B_{zz}
\end{pmatrix}. \\
&= \begin{pmatrix}
\lambda_1 & 0 & 0 \\
0 & \lambda_2 & 0 \\
0 & 0 & \lambda_3
\end{pmatrix}
\end{align*}

In free space the magnetic gradient tensor is both symmetric, so that the off-diagonal terms occur in equal pairs, and traceless, so that the sum of the diagonal terms is equal to zero (i.e., \(B_{xx} + B_{yy} + B_{zz} = 0\)). Therefore, there are five independent components of the tensor at each observation point.

Invariants quantities may be calculated from the magnetic gradient tensor that are able to resolve the direction to source as discussed in the next section. The advantages of gradiometry in general and of measuring tensor components rather than total field gradiometry have been reviewed by Schmidt and Clark (2006). New gradiometer systems are currently under development which will lead to the routine measurement of the magnetic gradient tensor both above ground and down-hole for targeting magnetic sources.

Eigen-analysis and invariants

The eigenvalues of (3) may be obtained by solving the characteristic equation

\[
\det \begin{pmatrix}
B_{xx} - \lambda_1 & B_{xy} & B_{xz} \\
B_{yx} & B_{yy} - \lambda_2 & B_{yz} \\
B_{zx} & B_{zy} & B_{zz} - \lambda_3
\end{pmatrix} = 0,
\]

with cubic roots \(\lambda_1, \lambda_2, \lambda_3\). These eigenvalues are calculable in this way regardless of the reference frame of the measurement and are so called rotational invariants (Pedersen and Rasmussen, 1990). A useful combination of these eigenvalues which remains invariant is

\[
\mu = \sqrt{-\lambda_2 - \lambda_1 \lambda_3}.
\]

where \(\lambda_1 \geq \lambda_2 \geq \lambda_3\) (Wilson, 1985). \(\mu\) is known as the scaled moment or normalised source strength. It is a scalar value that attains its maximum value directly over a dipole source and a number of other magnetic source geometries, regardless of the
magnetisation direction (Wilson, 1985; Schmidt et al., 2004; Clark, 2012). It is therefore a very useful quantity for targeting a magnetic source.

**Direction to source**

We follow the eigen-analyses of Schmidt et al. (2004) whereby a single measurement of the magnetic gradient tensor provides the vector from the magnetic source to the observation point (and the magnetic moment direction) with a four fold ambiguity. The general solution whereby there is one real solution and three “ghost” solutions is reproduced below, and the solutions for the degenerate cases where two eigenvalues are equal are given in Schmidt et al. (2004).

The eigenvectors \( \mathbf{v} \) corresponding to the eigenvalues \( \lambda = [\lambda_1, \lambda_2, \lambda_3] \) satisfy \( \Gamma \mathbf{v} = \lambda \mathbf{v} \). The unit vector in the direction of the two eigenvectors corresponding to the two eigenvalues with the largest absolute values is denoted \( \mathbf{v}_1 \) and \( \mathbf{v}_2 \). The two angles \( \phi \) and \( \theta \) are defined as

\[
\phi = \arccos \left( \frac{\lambda_3}{\mu} \right), \quad (8)
\]

and

\[
\theta = \arccos \left( \frac{\sin \phi}{\sqrt{\frac{\lambda_1}{\mu} + 2 \cos \phi} + \sin \phi^2} \right), \quad (9)
\]

respectively. Then the four possible directions from the source to the observation point are:

\[
\hat{n}_1 = \cos \theta \hat{v}_1 + \sin \theta \hat{v}_2, \quad (10)
\]

\[
\hat{n}_2 = -\hat{n}_1, \quad (11)
\]

\[
\hat{n}_3 = \cos \theta \hat{v}_1 - \sin \theta \hat{v}_2, \quad (12)
\]

\[
\hat{n}_4 = -\hat{n}_3. \quad (13)
\]

Two solutions can normally be dismissed straight away as they occur above the horizontal measurement plane when the magnetic gradient tensor is measured over the surface. When the observation points are constrained to a 1D borehole it is not so straightforward to automatically dismiss solutions. Triangulation is required at multiple observation points to resolve convergence and dismiss the ghosts.

To search for converging solutions a search grid \( s(x, y, z) \) is defined around the borehole observation string \( b(x, y, z) \) with the unit vector from the search grid location to the borehole point defined as

\[
\hat{r} = \frac{r}{|r|}, \quad (14)
\]

\[
= \mathbf{h} - \mathbf{s}. \quad (15)
\]

The minimum angle between \( \hat{r} \) and \( \hat{n}_i \), where \( i \) indexes the 4 possible solutions, is defined as

\[
\gamma = \min[\arccos (\hat{r} \cdot \hat{n}_i)], \quad (16)
\]

so that \( \gamma = 0 \) corresponds to a parallel alignment of one of the solutions with the vector from a search point to a borehole observation point. \( \gamma \) is computed at all points in the search grid \( s(x, y, z) \), summed over either part of or the whole borehole observation string, and normalised by \( \mu \) at that depth. Hence, minimising this metric searches for 3D regions of strong direction to source alignment and/or depths regions at which \( \mu \) peaks.

**Testing**

For all synthetic tests below we define a search grid from (minimum extent: step size: maximum extent) \( x = -300 : 10 : 300 \) m (East), \( y = -300 : 10 : 500 \) m (North), and \( z = -300 : 10 : 0 \) m (Depth). The IGRF is defined as \(-60^\circ\) inclination, \(0^\circ\) declination, and 60000 nT for the field strength. The magnetic gradient tensor components are calculated from the model bodies in the grid along a string of observation points defining a borehole located at \((x, y) = (0, 200)\) m orientated vertically. The solutions for the direction to the modelled source are calculated in the 3D search grid from either a subset or all of these observations points and plotted in 2D or 3D.

The colour scales for the plots are in arbitrary units where the warmer colours in the contour plots and the less transparent colour shells in the 3D isoplots represent a stronger solution (i.e., smaller summed values of the \( \gamma \) terms). All 3D isoplots are created in Leapfrog with the \(x, y\), and \(z\) axes plotted in red, green, and blue, respectively, with the borehole location indicated. The synthetic modelled bodies are overplotted in the 3D isoplots for reference.

**Single dipole source**

A dipole source is centred on \((x, y, z) = (0, 0, -200)\) m with radius \(r = 50\) m and susceptibility \(\chi = 0.01\) SI. The \(\gamma\) terms are summed over all points along the borehole observation string with the results plotted in Figures 1(top) and (bottom) as contour slices through the centre of the dipole and in Figure 2 as a 3D isosurface plot.
Direction to source tensor analysis

Two dipole sources

Two dipoles are now located either side of the borehole. The first is deep and centred on \((x, y, z) = (100, 100, -200)\) m with radius \(r = 50\) m and susceptibility \(\chi = 0.01\) SI. The second is shallow and centred on \((x, y, z) = (-100, 100, 50)\) m with radius \(r = 50\) m and susceptibility \(\chi = 0.01\) SI.

With multiple bodies in the search grid, summing over the entire observation string only vaguely shows the whereabouts of the source bodies. This is because as we move our observations down-hole, the measurements will first be dominated by the gradients of the shallow body, and then later by the gradients of the deeper body. The solutions are better analysed at a few observation depths over smaller strings of solutions. Figure 3 shows contour slices of the search grid through the centre of the dipoles at three depths along with the locations of the modelled source bodies.

Figure 3 a) and c) show the direction to the source nearest the respective observation points by vectoring towards the grid minimum. The vectors drawn with black lines pass directly through the source bodies. Due to the field contributions from both bodies the grid minima are pulled away from the centre of magnetisation of the closest source body, unlike Figure 1 where the minimum occurs at the centre of magnetisation. However, it remains a useful metric for targeting the bodies individually assuming the observation point is not too close to both bodies as Figure 3 b) illustrates.

Plunging sheet

A plunging sheet is now located South-West of the borehole. It is dipping away from the hole at 45° to the South-West with a strike azimuth of 150°. The thickness of the sheet is 30 m, with a depth to the top of 100 m, a depth extent of 100 m, a strike length of 100 m, and a susceptibility of \(\chi = 0.01\) SI. The situation modelled here may represent a missed drill attempt to intersect the face of the plunging sheet, for example.

Figure 4 shows the strongest solutions, when the \(\gamma\) terms are summed over the whole borehole string, occurring near the top of the sheet nearest the borehole and draping away. The measured gradients are likely to be dominated by the top edges of this sheet nearest the hole. This effect is reflected in the effective targeting in this plot since most vectors drawn from a point on the observation string to the grid minimum pass close to this region.

Figures 5, 6, and 7 plot the results when the observation string is analysed as three smaller strings over the top, middle, and bottom of the borehole, respectively. The isoplot shell colours correspond to the colour of the borehole segment analysed.

In the first of these plots, the observation string sees the target almost directly down-dip and therefore as a small surface area. The source direction solutions are mostly confined to the top edges where the largest gradients are generated. A vector from any part of the red observation string will likely graze one of the sheet’s edges or faces as it passes toward the grid minimum.

In the second of these plots, the observation string runs along side the depth extend of the target body. Again, the gradients are dominated by the target’s edges at closest approach, but a vector from any part of the blue observation string to the grid minimum.

Figure 2: A 3D isosurface plot showing the calculated source location in the search grid for a single dipole source. The modelled source body is plotted as a transparent blue sphere.

Figure 3: Contour slices through \(y = 100\) m for three different observation strings consisting of four points. The white circles show the locations of the two dipoles and the black arrows show the direction from the observation string to the strongest source solution or solutions. a) At the start of the borehole \(z = 0\) m. b) Between the two bodies \(z = 100\) m. c) At the end of the hole \(z = 295\) m.
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Figure 4: A 3D isosurface plot showing the calculated source location in the search grid summed over the entire borehole observation string. The plunging sheet body is shown in yellow.

Figure 5: A 3D isosurface plot showing the calculated source location in the search grid summed over the first 100 m of the borehole (red line segment). The plunging sheet body is shown in yellow.

Figure 6: A 3D isosurface plot showing the calculated source location in the search grid summed over the 100-200 m segment of the borehole (blue line segment). The plunging sheet body is shown in yellow.

Figure 7: A 3D isosurface plot showing the calculated source location in the search grid summed over the 200-300 m segment of the borehole (purple line segment). The plunging sheet body is shown in yellow.

In the third of these plots, the observation string is far from the target and the field will appear more dipole like. Any point along the purple observation string is roughly equidistant from the bodies top and bottom edges. Here then, the vectors to the grid minima are well constrained to pass directly through the middle of the target.

CONCLUSIONS

Eigen-analyses similar to Schmidt et al. (2004) have been applied to a string of down-hole measurements with search methods described for calculating the estimated direction to source. The methods work extremely well for the ideal situation of an isolated dipole source. Although the magnetisation for the dipole example investigated here is purely induced, very similar results are calculated for a dipole with an arbitrary remanent component.

When dealing with multiple bodies or non-dipole geometries the magnetic tensor data string should be post-analysed as a series of shorter observation strings to help constrain the most likely direction to magnetic source.

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